LUND UNIVERSITY

Structural Optimization@LTH

Mathias Wallin
Lund University, Solid mechanics

Computational mechanics@LTH

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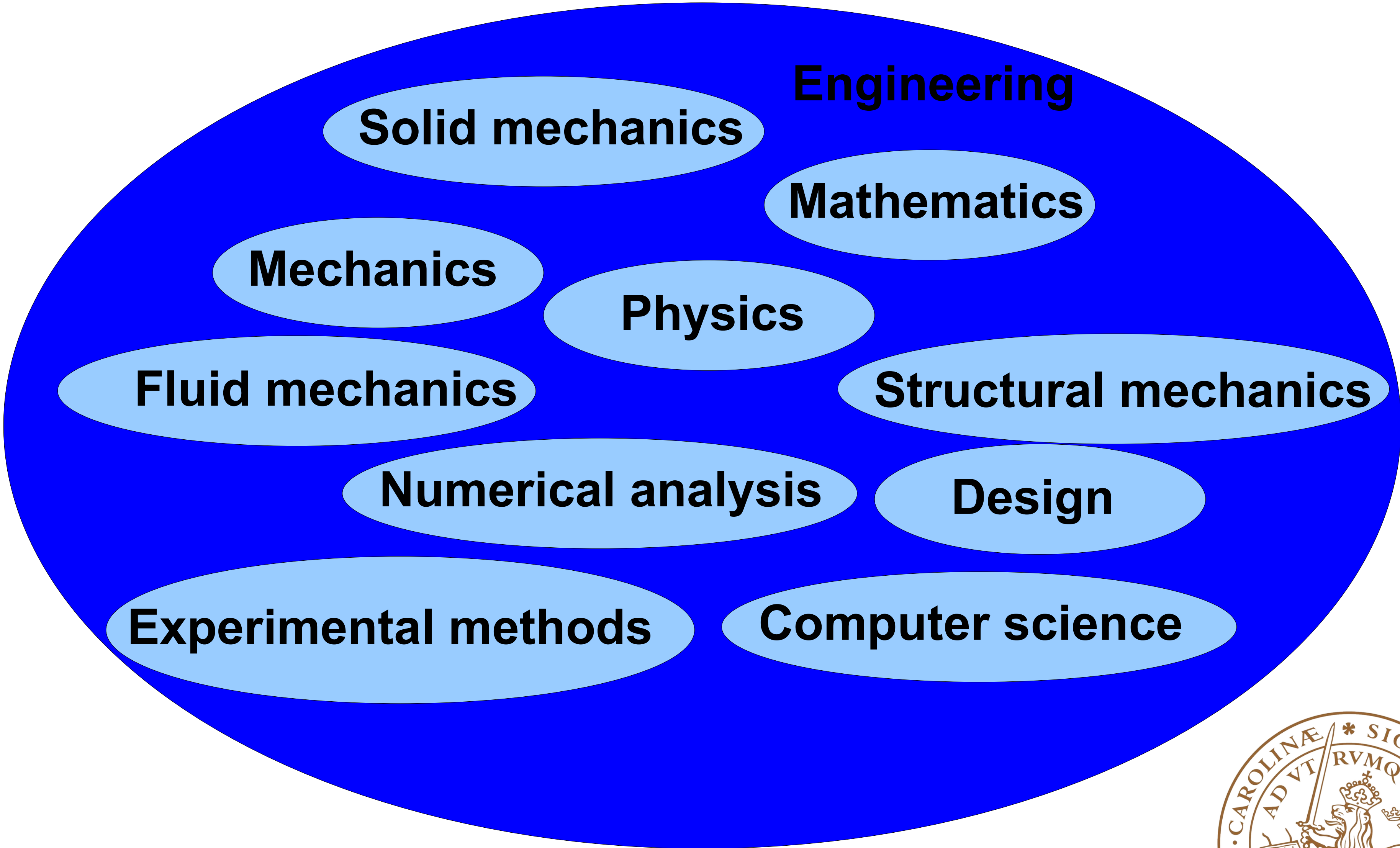
Lund University, Solid mechanics

What is computational mechanics ?

- Solid mechanics
- Mathematics
- Mechanics
- Physics
- Structural mechanics
- Fluid mechanics
- Numerical analysis
- Design
- Experimental methods
- Computer science

What is computational mechanics ?



Strategies for teaching computational mechanics ?

- **Software oriented**

SIMULIA ABAQUS

Ansys

LS-DYNA®

ALTAIR

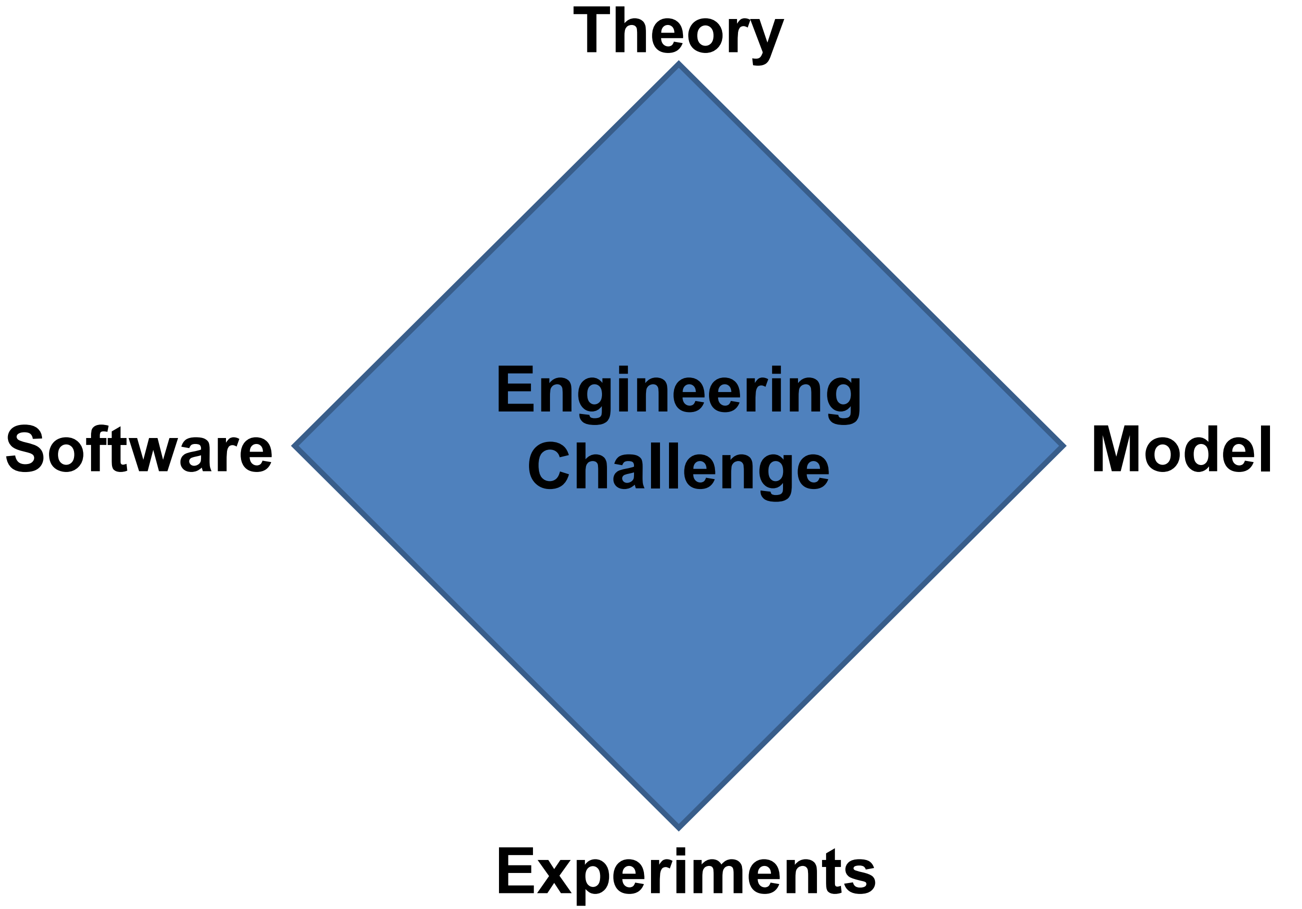
Strategies for teaching computational mechanics ?

- Software oriented
- Mathematical oriented

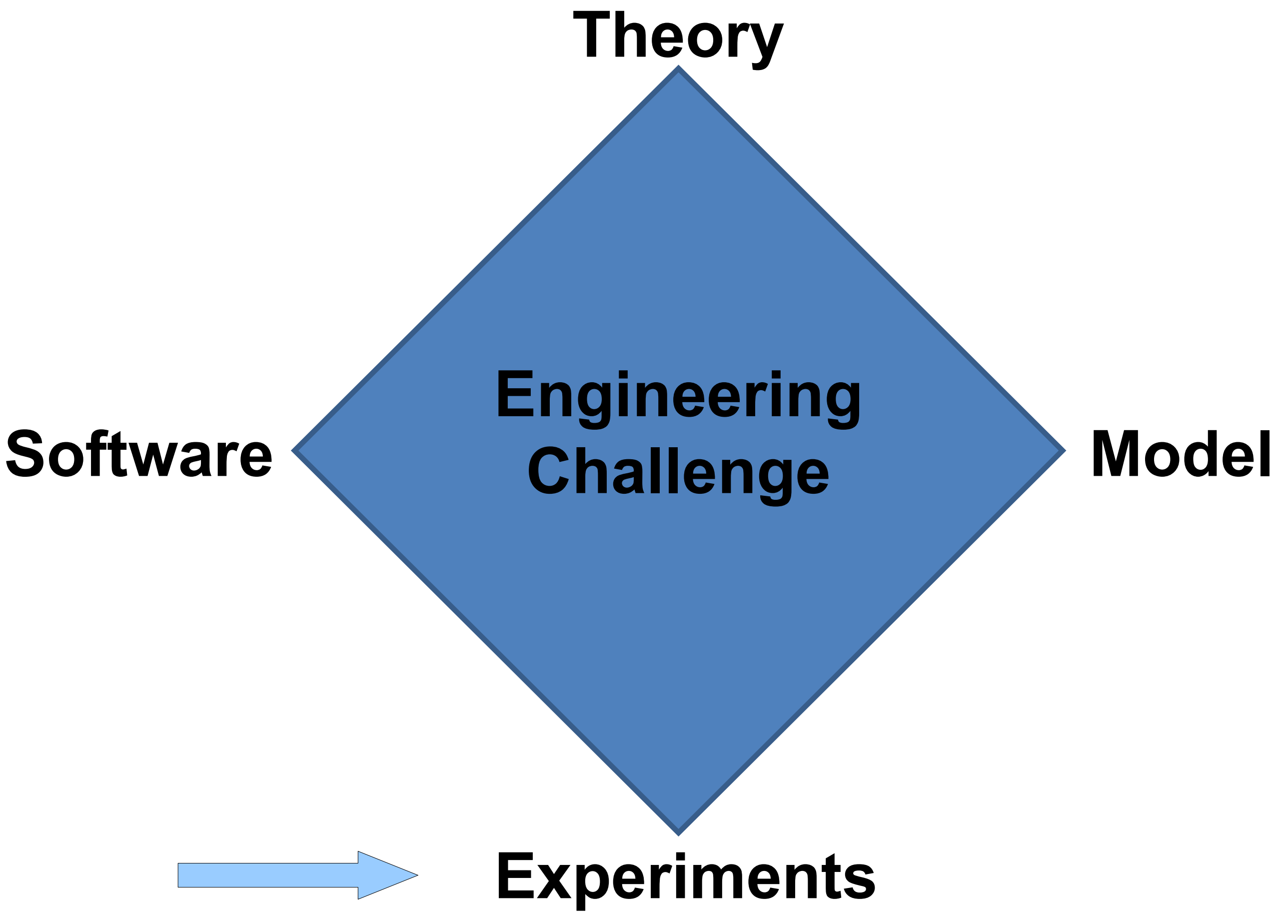
Strategies for teaching computational mechanics ?

- Software oriented
- Mathematical oriented
- Engineering oriented

The engineering strategy



The engineering strategy



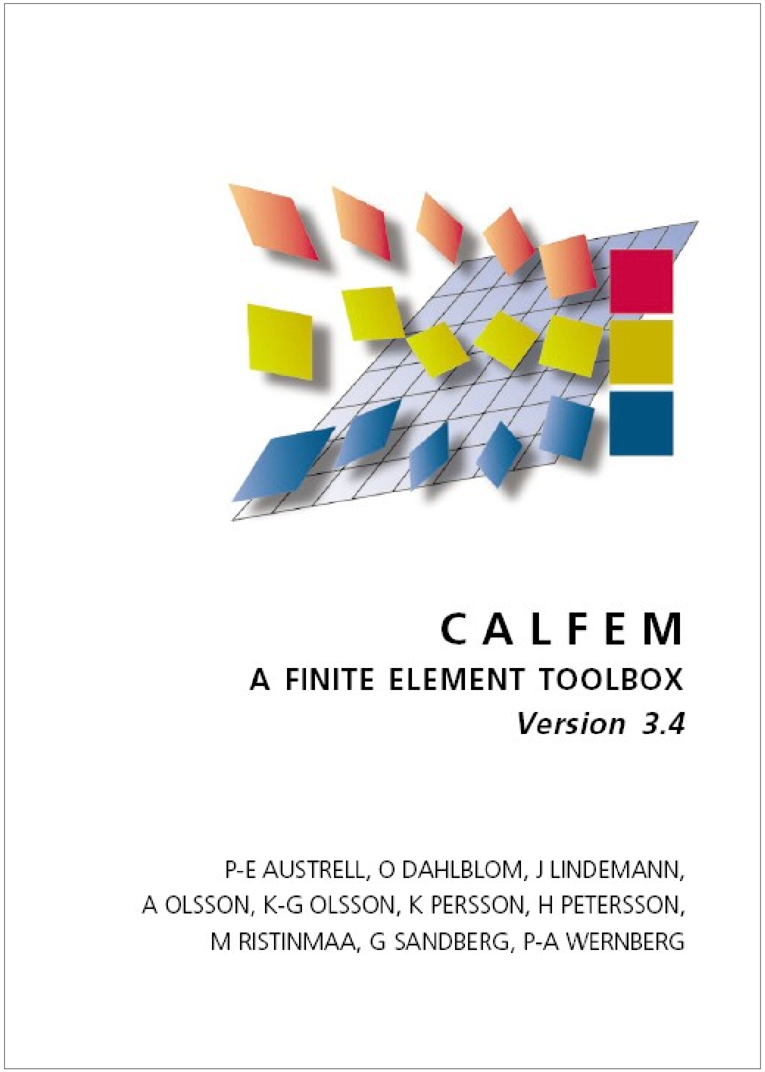
Target students

- Mechanical Engineering
- Civil Engineering
- Engineering Physics
- Engineering Mathematics

Motivation

- **We want to teach the generic knowledge, the mathematics behind the software and the underlying principles**
- **We want to avoid using streamlined software packages as ”black boxes”**
- **We want to promote understanding rather than knowledge of a specific software, confidence in performing simulations and the skills that allow critical reviewing of the results**

CALFEM



- Software (matlab toolbox)
- Manual
  - Introduction
  - Syntax of subroutine calls
  - Link to theory
  - Examples

CALFEM

Contents

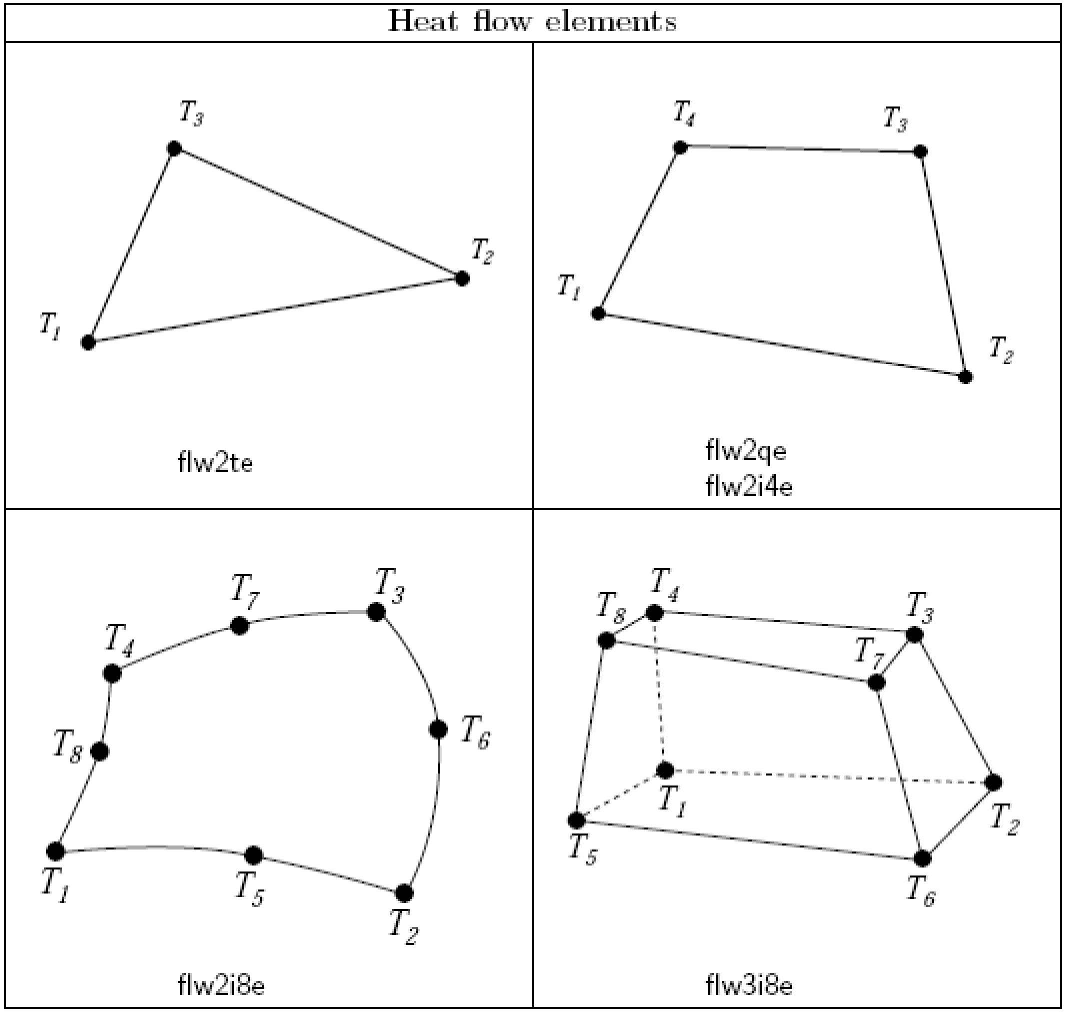
CALFEM

Contents

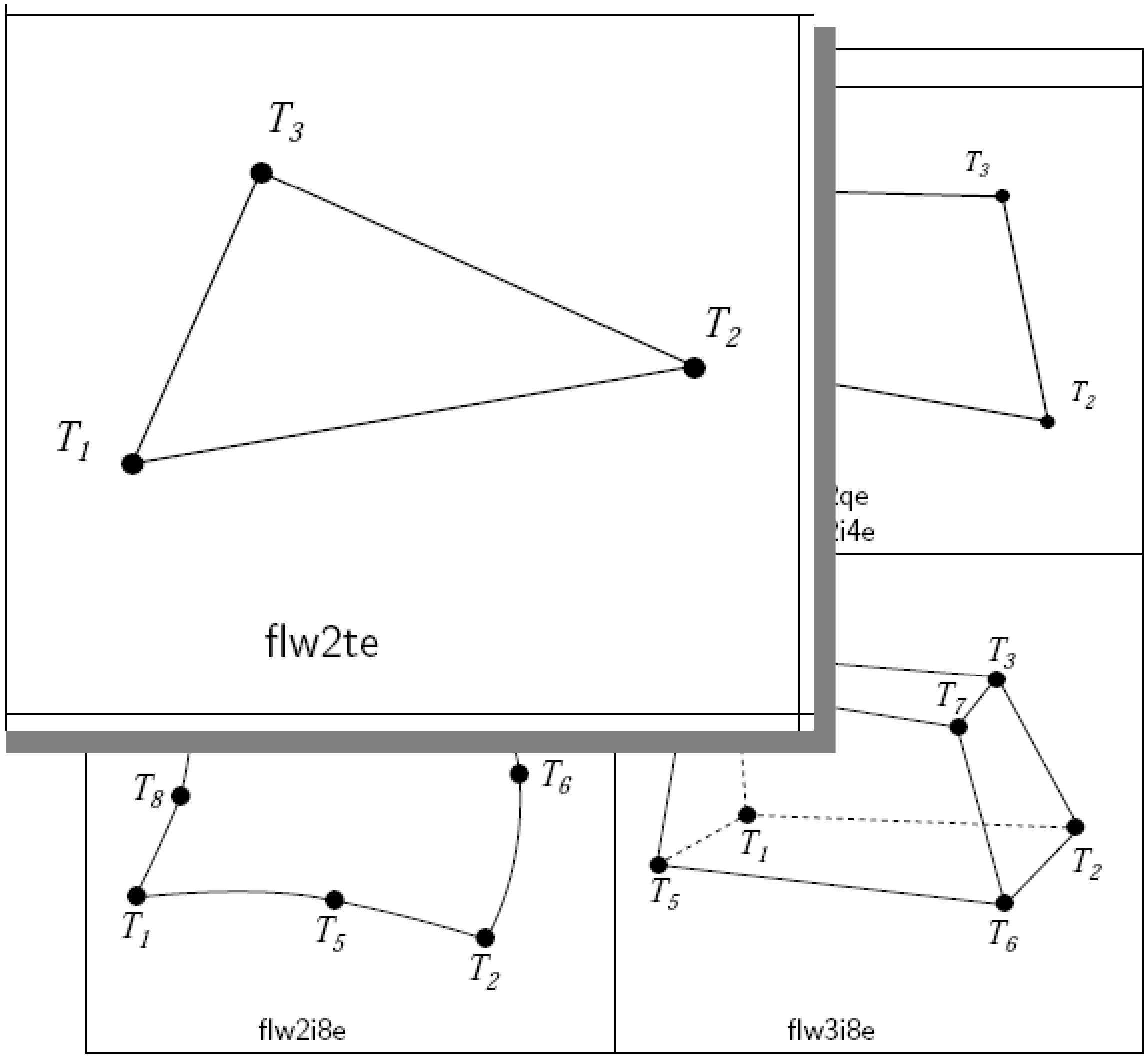
CALFEM

Problem type	a	D	$\mathbf{f}_l$	Designation
Heat flow	T	λ_x , λ_y	Q	T = temperature λ_x , λ_y = thermal conductivity Q = heat supply
Groundwater flow	ϕ	k_x , k_y,	Q	ϕ = piezometric head k_x, k_y = permeabilities Q = fluid supply
St. Venant torsion	ϕ	$\frac{1}{G_{zy}}$, $\frac{1}{G_{zx}}$	2Θ	ϕ = stress function G_{zy}, G_{zx} = shear moduli Θ = angle of torsion per unit length

CALFEM



CALFEM

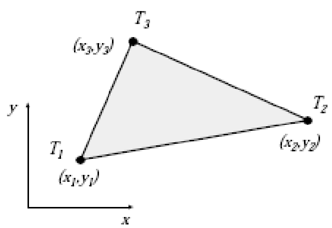


T3
T2
T1
flw2te
T3
T2
2qe
2i4e
T8
T6
T1
T5
T2
flw2i8e
T3
T7
T1
T2
T5
T6
flw3i8e

CALFEM

Purpose:

Compute element stiffness matrix for a triangular heat flow element.



Syntax:

```
Ke=flw2te(ex,ey,ep,D)
[Ke,fe]=flw2te(ex,ey,ep,D,eq)
```

Description:

flw2te provides the element stiffness (conductivity) matrix Ke and the element load vector fe for a triangular heat flow element.

The element nodal coordinates x_1, y_1, x_2 etc, are supplied to the function by ex and ey, the element thickness t is supplied by ep and the thermal conductivities (or corresponding quantities) k_{xx}, k_{xy} etc are supplied by D.

$$\text{ex} = [\, x_1 \;\; x_2 \;\; x_3 \,] \qquad \text{ep} = [\, t \,] \qquad \text{D} = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}$$
$$\text{ey} = [\, y_1 \;\; y_2 \;\; y_3 \,]$$

If the scalar variable eq is given in the function, the element load vector fe is computed, using

$$\text{eq} = [\, Q \,]$$

where Q is the heat supply per unit volume.

Theory:

The element stiffness matrix $\mathbf{K}^e$ and the element load vector $\mathbf{f}_l^e$, stored in Ke and fe, respectively, are computed according to

$$\mathbf{K}^e = (\mathbf{C}^{-1})^T \int_A \bar{\mathbf{B}}^T \, \mathbf{D} \, \bar{\mathbf{B}} \; t \; dA \;\; \mathbf{C}^{-1}$$
$$\mathbf{f}_l^e = (\mathbf{C}^{-1})^T \int_A \bar{\mathbf{N}}^T \, Q \; t \; dA$$

with the constitutive matrix $\mathbf{D}$ defined by D.

The evaluation of the integrals for the triangular element is based on the linear temperature approximation $T(x,y)$ and is expressed in terms of the nodal variables T_1, T_2 and T_3 as

$$T(x,y) = \mathbf{N}^e \mathbf{a}^e = \bar{\mathbf{N}} \, \mathbf{C}^{-1} \mathbf{a}^e$$

where

$$\bar{\mathbf{N}} = [\, 1 \;\; x \;\; y \,] \qquad \mathbf{C} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \qquad \mathbf{a}^e = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

and hence it follows that

$$\bar{\mathbf{B}} = \nabla \bar{\mathbf{N}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix}$$

Evaluation of the integrals for the triangular element yields

$$\mathbf{K}^e = (\mathbf{C}^{-1})^T \, \bar{\mathbf{B}}^T \, \mathbf{D} \, \bar{\mathbf{B}} \, \mathbf{C}^{-1} \, t \, A$$

$$\mathbf{f}_l^e = \frac{QAt}{3} [\, 1 \;\; 1 \;\; 1 \,]^T$$

where the element area A is determined as

$$A = \frac{1}{2} \det \mathbf{C}$$

CALFEM based courses

Finite Element method

Structural optimization

Computational inelasticity

Nonlinear finite element

Structural dynamic computing

CALFEM based courses

- Finite Element method
- Structural optimization
- Computational inelasticity
- Nonlinear finite element
- Structural dynamic computing
- Modern experimental methods

CALFEM based courses

Finite Element method

Structural optimization

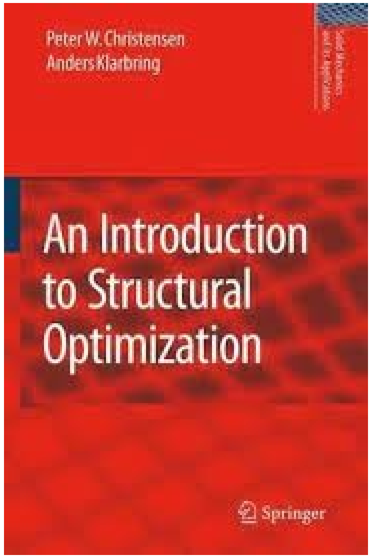
Computational inelasticity

Nonlinear finite element

Structural dynamic computing

Modern experimental methods

Structural optimization



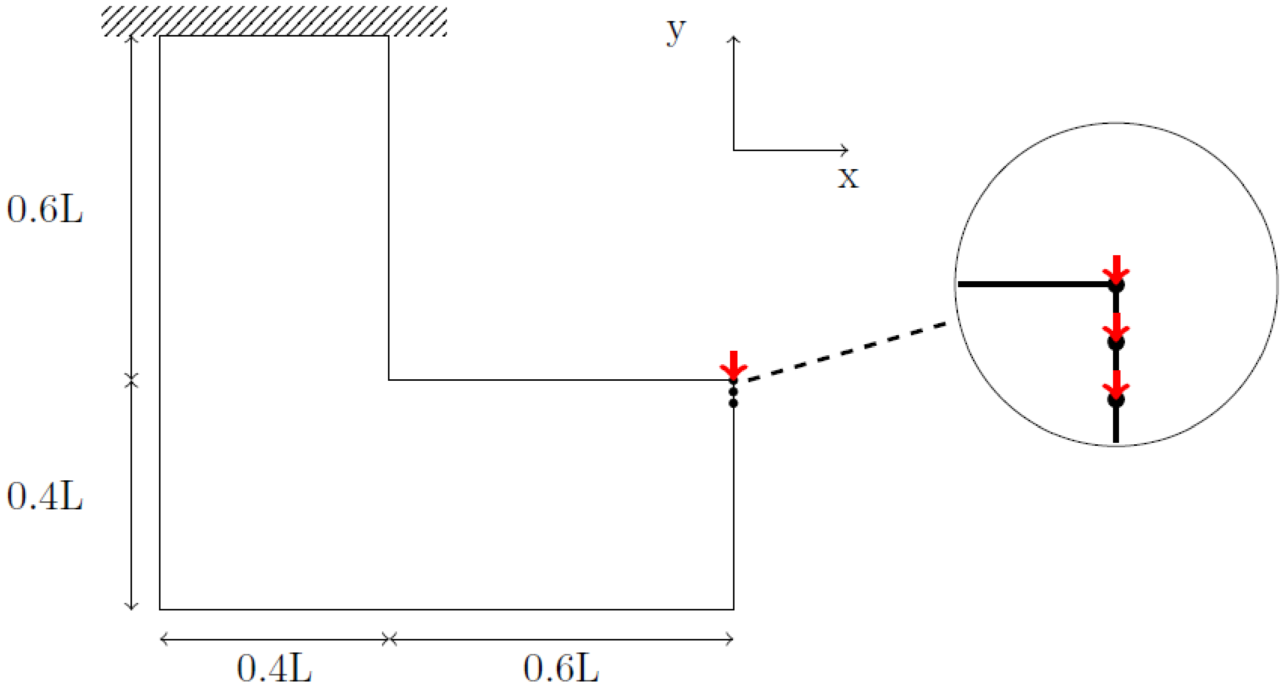
+ An Introduction to Structural Optimization

+ Additional literature

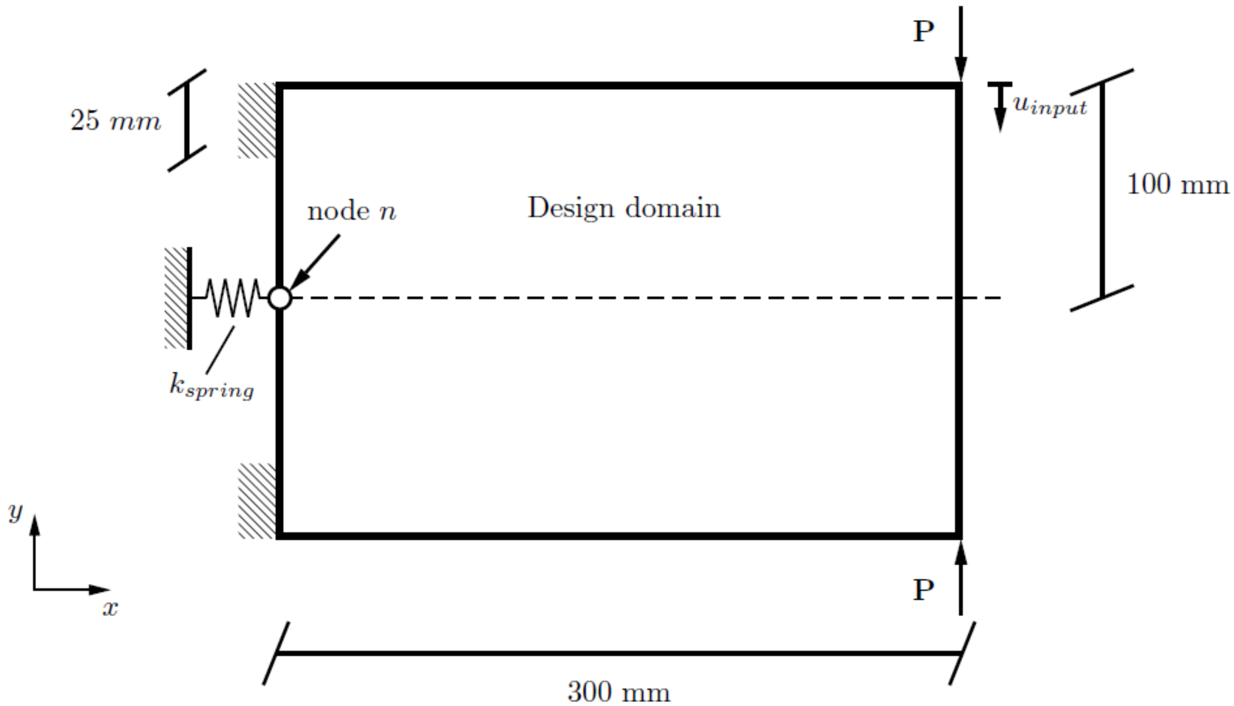
+ Journal papers

+ 20-30 students annually

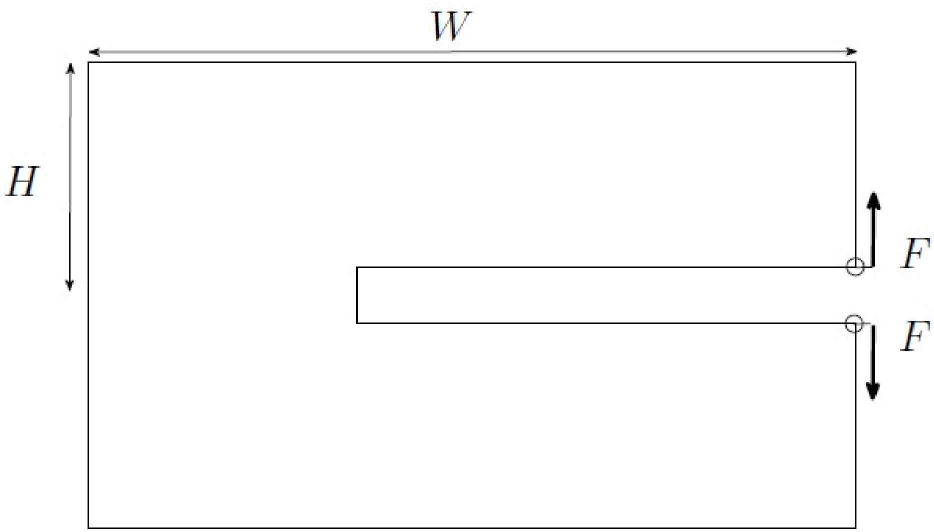
Structural optimization- stress constraints



Structural optimization-compliant mechanism



Structural optimization - filter influence



+ Sensitivity filter

+ Density filter

+ PDE filter

Masters projects

Computationally Efficient Methods in Topology Optimization, Vilmer Dahlberg

Topology optimization of transient thermo-mechanical problems using multiple materials, Olov Gunther-Hansen

Isogeometric analysis as a tool for large deformation shape optimization, Max Kurki

Shape optimization for large deformations using B-splines. Filip Sjövall

Topology optimization: perimeter restriction using total variation. Jonas Fredriksson

Stress constrained topology optimization for non-proportional loading, Gunnar Granlund

PhD projects

Reduced order modeling in Topology optimization, Vilmer Dahlberg

Topology optimization of electro-activated structures, Daniel Hård

Topology optimization of transient heat conduction problems, Gunnar Granlund

Shape optimization of non-linear elastic problems, Filip Sjövall

Eigenvalue constrained topology optimization problems, Anna Dalklint